The City as a System for Innovation and Entrepreneurship

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Provided that some groups on earth continue either muddling or revolutionizing themselves into periods of economic development, we can be absolutely sure of a few things about future cities:

The cities will not be smaller, simpler or more specialized as cities of today.

Rather, they will be more intricate, comprehensive, diversified and larger than today’s and will have even more complicated jumbles of old and new things as ours do.

Jane Jacobs
The Economy of Cities, 1980
Per capita GDP and urbanization

1 Definition of urbanization varies by country; pre-1950 figures for the United Kingdom are estimated.  
2 Historical per capita GDP series expressed in 1990 Geary-Khamis dollars, which reflect PPP.  

SOURCE: Population Division of the United Nations; Angus Maddison via Timetrics; Global Insight; Census reports of England and Wales; Honda in Steckel & Floud, 1997; Bairoch, 1975
[Real cities, present] situations in which several dozen quantities are all varying simultaneously and in subtly connected ways

Jane Jacobs
The death and life of great American cities 1961
Wealth Creation is driven by Innovation


$\beta=1.15$ (95% C.I. = [1.11, 1.18])

adjusted $R^2=0.89$

Data courtesy of Richard Florida and Kevin Stolarick.
A. GDP

B. Urbanized Area

C. Employment

D. Patents

\( \beta = 1 + \frac{1}{6} \)

\( \beta = 1 - \frac{1}{6} \)

Bettencourt + Lobo (2015)

Urban Scaling in Europe
Infrastructure & socioeconomic rates

Volume of Infrastructure

\[ N^{\beta_i} \]

\[ \beta_i = 1 - \delta \]

Social Outputs

\[ N^{\beta_s} \]

\[ \beta_s = 1 + \delta \]

\[ \delta \approx 0.15 \]
The Origins of Scaling in Cities

Luís M. A. Bettencourt

Despite the increasing importance of cities in human societies, our ability to understand them scientifically and manage them in practice has remained limited. The greatest difficulties to any scientific approach to cities have resulted from their many interdependent facets, as social, economic, infrastructural, and spatial complex systems that exist in similar but changing forms over a huge range of scales. Here, I show how all cities may evolve according to a small set of basic principles that operate locally. A theoretical framework was developed to predict the average social, spatial, and infrastructural properties of cities as a set of scaling relations that apply to all urban systems. Confirmation of these predictions was observed for thousands of cities worldwide, from many urban systems at different levels of development. Measures of urban efficiency, capturing the balance between socioeconomic outputs and infrastructural costs, were shown to be independent of city size and might be a useful means to evaluate urban planning strategies.

Cities exist, in recognizable but changing forms, over an enormous range of scales (1), from small towns with just a few form rather than function, which limit their ability to help us understand and plan cities.

Recently, our increasing ability to collect and...
Cities are co-located social networks in space and time.
Individuals actively integrate possibilities of the urban environment.

wealth
health
education
well-being
violence
innovation
...

[Diagram of a grid with various elements connected]
## Predictions and Consequences

### Urban Scaling Relations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model (D=2,H=1)</th>
<th>Model (D, H)</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land area $A$</td>
<td>$A = a N^\alpha$</td>
<td>$\alpha = \frac{D}{D+H}$</td>
<td>spatial densification</td>
</tr>
<tr>
<td>Network volume $A_n$</td>
<td>$A_n = A_0 N^\nu$</td>
<td>$\nu = D + DH - H$</td>
<td>growth of infrastructure</td>
</tr>
<tr>
<td>Network length $L$</td>
<td>$L = L_0 N^{\lambda}$</td>
<td>$\lambda = \alpha$</td>
<td>area filling networks</td>
</tr>
<tr>
<td>Average network width $\bar{S}$</td>
<td>$\bar{S} = S_0 N^{\bar{\sigma}}$</td>
<td>$\bar{\sigma} = 1 - \delta$</td>
<td>widening of roads</td>
</tr>
<tr>
<td>Interactions per capita $y$</td>
<td>$y = Y_0 N^\delta$</td>
<td>$\delta = \frac{D}{D+H}$</td>
<td>increased interactions</td>
</tr>
<tr>
<td>Socioeconomic rates $Y$</td>
<td>$Y = Y_0 N^\beta$</td>
<td>$\beta = 1 + \delta$</td>
<td>acceleration of social rates</td>
</tr>
<tr>
<td>Power dissipation $W$</td>
<td>$W = W_0 N^{\omega}$</td>
<td>$\omega = 1 + \delta$</td>
<td>increased congestion</td>
</tr>
<tr>
<td>Land Value $P_L$</td>
<td>$P_L = P_0 N^{\delta_L}$</td>
<td>$\delta_L = \alpha - \delta$</td>
<td>increased land rents</td>
</tr>
</tbody>
</table>

### Table 1: Urban indicators and their scaling relations. The first column shows expected mean-field values for scaling exponents vs. population size ($D=2, H=1$). The second column shows the value of scaling quantities in general spatial dimensions. The third column describes the effect.

### Figure 1: Scaling of urban infrastructural and socioeconomic quantities. (a) Total lane miles (volume) of roads in US metropolitan areas in 2006 (blue dots). Lines show the best fit to a scaling relation $Y = Y_0 N^\beta$ with $\beta = 1.126 \pm 0.023$ (95% CI, $R^2 = 0.96$), the theoretical prediction, $\beta = 7/6$ (yellow), and linear scaling $\beta = 1$ (black). (b) Gross Metropolitan Product of US metropolitan area in 2006 (green dots). Lines show the best fit (red), with $\beta = 1.126 \pm 0.023$ (95% confidence interval, $R^2 = 0.96$), the theoretical prediction, $\beta = 7/6$ (yellow), and proportional scaling, $\beta = 1$ (black).

### Social Benefits - Costs

$$Y^* - W^*$$

- $G_{\text{min}}$: city exists
- $G^*$: city exists
- $G_{\text{max}}$: city unstable

### Scaling and Interdependence of Social, Economic, Infrastructural and Geographic factors

- $G_{\text{min}}$: city exists
- $G^*$: city exists
- $G_{\text{max}}$: city unstable
Deviations from Scaling

**Scale Adjusted Metropolitan Indicators: SAMIs**

\[ \xi_i(t) = \ln \frac{Y_i(t)}{Y_0 N^\beta(t)} \]

What is the structure of each city’s deviation?

What is its local flavor ...?
Ranking Cities independently of their population size
Ranking Cities
independently of their population size

1. Corvallis OR
2. Burlington VT
3. San Jose CA
4. Boise City ID
5. Kokomo IN
38. San Francisco CA
79. Boston MA
163. Dallas TX
179. Denver CO
185. Los Angeles CA
253. New York NY
336. Merced CA
337. Yuma AZ
338. Visalia CA
339. Shreveport LA
340. McAllen TX
Temporal persistence

personal income
Temporal persistence
Temporal persistence of scaling deviations

Persistence times are in the order of a few decades!
Beyond population size are there correlations between

Population size accounts for most of the (co)-variation in these quantities. Cross correlations explain only 5-15% of the variation.
Professional Diversity and Classification Resolution

Occupations in US Metropolitan Statistical Areas

A good fit at all resolutions:

\[ D(N_e) = d_0 \frac{\left( \frac{N_e}{N_0} \right)^\gamma}{1 + \left( \frac{N_e}{N_0} \right)^\gamma}. \]
The limit of infinite resolution

\[ D(N) = d_0h \left( \frac{N}{N_0} \right) \left( \frac{N}{N_0} \right) \gamma \rightarrow \begin{cases} D_0N^\gamma, & N << N_0, \\ d_0(r), & N >> N_0, \end{cases} \]

In the limit:

\[ \frac{N}{N_0} \rightarrow 0; \quad h \rightarrow 1, \quad D_0 \rightarrow \frac{d_0}{N_0^\gamma} \]

In the limit:

\[ \frac{N}{N_0} \rightarrow +\infty; \quad h \rightarrow \left( \frac{N_0}{N} \right)^\gamma, \]

A scaling limit exists iff:

\[ D_0 \rightarrow \frac{d_0}{N_0^\gamma} = \text{const.} \quad \text{with} \quad D(N) = D_0N^\gamma. \]
The rank size distribution of professions

From D(N), for all N, derive frequency distribution

\[ f(i) = \frac{N_e}{N_0} \left( \frac{d_0 - i}{i} \right)^{1/\gamma}. \]

\[ p(i) = \frac{f(i)}{\sum_{j=1}^{D(N)} f(j)} = \frac{1 - \gamma}{\gamma} \frac{i^{-1/\gamma}}{1 - D(N)^{-\frac{1-\gamma}{\gamma}}}; \]

Indices of Diversity:

\[ HH(N) = \sum_{i=1}^{D(N)} p^2(i) = \frac{\delta^2}{1 - \delta^2} \frac{1 - D_0^{-\frac{1+\delta}{1-\delta}} N^{-1-\delta}}{(1 - D_0^{\frac{\delta}{1-\delta}} N^{-\delta})^2} \]

\[ \approx \frac{\delta^2}{1 - \delta^2} \left( 1 + \frac{2}{D_0^{\frac{\delta}{1-\delta}} N^\delta} \right). \]

\[ S = -\sum_{i=1}^{D(N)} p(i) \ln p(i) \approx \frac{1}{\delta} - D_0^{-\delta/(1-\delta)} N^{-\delta} \ln(D_0^{1/\gamma} N) \]

Universality!
Professional Diversity and Urban Productivity

Specialization and Division of Labor as sources of increases in urban productivity

\[ \mathcal{L}(d; \lambda) = \frac{g(kd)}{d} - \lambda (kd - A). \]

\[ d = \frac{A}{k} = \frac{A}{k_0} \frac{1}{N^\delta}, \quad w = \frac{g(A)}{A} k = \frac{g(A)}{A} k_0 N^\delta, \]

\[ \frac{dg}{dA} - \frac{g}{A} - \lambda_1 A = 0 \rightarrow g(A) = \left[ C + \int^A dA' \lambda_1(A') \right] A, \]

\[ D_0 = \frac{A}{k_0} = 0.05 \]

Bettencourt, Samaniego, Youn, 2013
Information, Connectivity & Productivity Growth

A. Redundancy

B. Diversity + Complementarity

C. Metcalfe’s Limit

Connected Phase

Disconnected Phase

\[ w(N), i(N) \]

\[ w_D, i_D \]

Bettencourt 2014
The role of “technology” in social networks